Sheet 3

1 (a) What is meant by the gradient of a scalar field ?

(b) Derive an expression for it to explain (a)

---

2 Show that the gradient of scalar field $f(x,y,z)$, $f(x,y,z) = x + y$ is normal to lines of constant f .

---

3 Determine the rate of change of the scalar field $f\ (x,y,z) = xy + 2z^2$ at p $(1,1,1)$ in the direction of the vector $\bar{a}_x - 2\bar{a}_y + \bar{a}_z$

---

4 Determine the gradient of the following scalar fields :

(a) $f = 5x + 10xz - xy + 6$
(b) $f = 2\sin\phi - rz + 4$
(c) $f = 2r\ \cos\theta - 5\phi + 2$

---

5 Consider the scalar field of the potential V is give by $V = 2x^2y - 5z$. Show that the vector field $\bar{F} = \nabla V$ is a conservative field or not along any closed path

---

6 Show that $\bar{F}(r,\theta,\phi) = \frac{k}{r^2}\ \bar{a}_r$ is a conservative field for any closed contour C .

---

7 Compute the line integral of $\bar{F} = \nabla f$ for $f = 2x + yz - xy$ from $P_1(1,-1,1)$ to $P_2(0,0,0)$ along paths consists of

(a) a straight line between P_1 and P_2

(b) straight line segments connecting P_1 to $(1,-1,0)$ to $(1,0,0)$ to P_2

Comment on your results .